Feedback Tether Deployment and Retrieval

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Introduction

SPACE tethers have been proposed for many important applications such as the study of planetary atmosphere, micro and variable-*g* experiments, space construction, and energy transmission. One of the primary issues in tether utilization is fast deployment/retrieval of attached payloads. Much work has been done on developing models for studying tether dynamics during deployment and retrieval.[1] These models include tether flexibility, atmospheric effects, orbital eccentricity, and planetary oblateness effects.

Two basic methods for controlling tether deployment/retrieval have emerged to date: 1) tension control[1-5] (reeling in or out) and 2) crawler mechanism.[6] The second method has been proposed due to concerns of slow retrieval rates in the terminal phases and violent tether oscillations during tension controlled retrieval. The disadvantage of this method is that if the tether is not retrieved it may have to be jettisoned. A comparison of the two aforementioned control schemes has been conducted by Glickman and Rybak.[7] Overall, much physical insight can be obtained by using simple mathematical models.[8]

A Lyapunov (mission function) approach has been used for tether deployment and retrieval by Fujii and Ishijima.[9] The proposed nonlinear tension control law has been designed for controlling deployment and retrieval in the orbital plane. It performs well, but terminal oscillations of the tether length and tension are encountered during deployment. Furthermore, the pitch angle seems to remain constant during the terminal phase of retrieval indicating that it cannot be controllable to the origin. In this Note, we consider a modified Lyapunov control law to eliminate the terminal oscillations observed by Fujii and Ishijima[9] and show that rapid retrieval is possible if moderate pitch angle excursion of the tether is allowed during the intermediate phase of retrieval. In all cases, the pitch angle can be controlled to the desired equilibrium point.

Formulation

We assume that the tether is of negligible mass and it remains straight. It has been shown in previous studies that including the mass of the tether in the dynamic model does not qualitatively change the behavior during deployment and retrieval. The equations of motion of the system[9] are

$$\ddot{l} - l[(\dot\Theta+\Omega)^2 + 3\Omega^2\cos^2\Theta] = -T/m \tag{1a}$$

$$\ddot\Theta + 2(\dot{l}/l)(\dot\Theta+\Omega) + 3\Omega^2\cos\Theta\sin\Theta = 0 \tag{1b}$$

where l indicates the instantaneous tether length, Θ, the pitch angle, Ω, the orbital rate, T, the tension, and m, the mass of the subsatellite. These equations can be nondimensionalized by defining the following nondimensional variables:

$$\tau = \Omega t, \quad \lambda = l/L, \quad \hat{T} = T/(m\Omega^2 L)$$

where L is the maximum tether length. The nondimensional equations are given by the following:

$$\lambda'' - \lambda(1+\Theta')^2 + \lambda - 3\lambda\cos^2\Theta = -\hat{T} \tag{2a}$$

$$\Theta'' + 2(\lambda'/\lambda)(1+\Theta') + 3\cos\Theta\sin\Theta = 0 \tag{2b}$$

where the prime symbol indicates the derivative with respect to nondimensional time. It is assumed from the onset that the out-of-plane motion is actively controlled.

We next consider a candidate Lyapunov function to determine a suitable feedback control law for the system. The following trial Lyapunov function, which is related to the Hamiltonian, is selected:

$$V = 1/2[\lambda'^2 + K_1(\lambda-\lambda_f)^2 + (K_2 + 3\lambda^2)$$
$$(1/3\,\Theta'^2 + \sin^2\Theta)] \tag{3}$$

where λ_f is the desired final value of λ. K_1 is a positive constant and K_2 can either be a positive constant or zero. This Lyapunov function is primarily quadratic; the quartic terms have been included to obtain mathematical simplifications. The time derivative of V is given by

$$V' = \lambda'[3\lambda - \hat{T} + K_1(\lambda-\lambda_f) - 2/3K_2\Theta'(1+\Theta')/\lambda] \tag{4}$$

We select

$$\hat{T} = 3\lambda + K_1(\lambda-\lambda_f)$$
$$- 2/3K_2\Theta'(1+\Theta')/\lambda + K_3\lambda', K_3 > 0 \tag{5}$$

as the nondimensional tension control law (K_3 is greater than zero), so that

$$V' = -K_3\lambda'^2 \tag{6}$$

To investigate the stability of the closed-loop system, let us assume that $\lambda' = 0$. From Eqs. (2) and (5) we obtain

$$\lambda[(1 + \Theta')^2 + 3\cos^2\Theta] = 4\lambda + K_1(\lambda-\lambda_f)$$
$$- 2/3K_2\Theta'(1 + \Theta')/\lambda \tag{7}$$

$$\Theta'' + 3\cos\Theta\sin\Theta = 0 \tag{8}$$

By differentiating Eq. (7) and substituting Eq. (8), we can see that since λ is a constant, Θ' must also be a constant. This leads to Θ being equal to zero, π, or $\pm\pi/2$. The equilibrium values at $\pm\pi/2$ are to be avoided. If we consider downward deployment and upward retrieval, then the equilibrium point $\lambda = \lambda_f$ and $\lambda' = \Theta = \Theta' = 0$ is appropriate.

In what follows, we consider deployment and retrieval of a tethered satellite from the Space Shuttle assumed to be in a circular orbit at an altitude of 220 km, with an orbital rate of 0.07068 rad/min. The tether is assumed to be 100 km long.

Deployment

The initial conditions for the motion of the tether are $\lambda = 0.01$, $\lambda' = 0.5$ (58.8 m/s), $\Theta = \Theta' = 0$, and the desired final conditions are $\lambda = 1.0$, $\lambda' = \Theta = \Theta' = 0$. The gains selected are $K_1 = 2.0$, $K_2 = 0$, and $K_3 = 4$. It is important to keep the tension positive. This requires a few trials to select the gains and the initial tether velocity. Figure 1 shows the nondimensional tether length, the pitch angle, and the tension. It can be seen that deployment is essentially complete in about one orbit. The nondimensional tension settles down to its equilibrium value of 3.

Retrieval

The preceding control law is also applicable for retrieval. The initial conditions are assumed to be $\lambda = 1.0$, $\lambda' = 0$,

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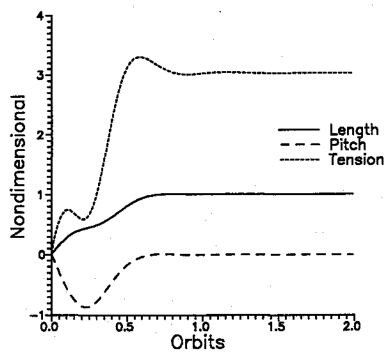

Fig. 1 Deployment using tension control.

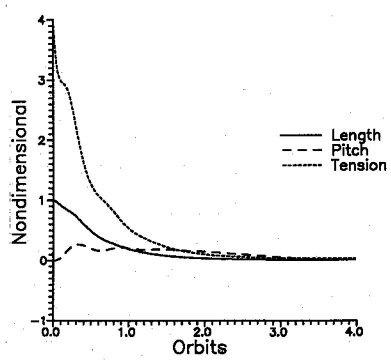

Fig. 2 Retrieval using tension control.

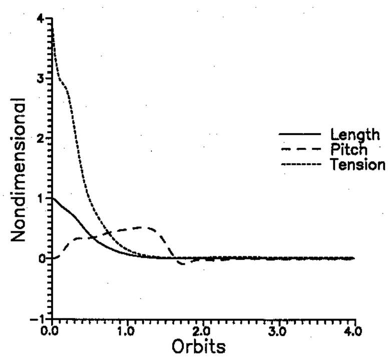

Fig. 3 Retrieval using tension control.

$\Theta = 0$, $\Theta' = 0$, and the desired final conditions are $\lambda = 0.01$, $\lambda' = \Theta = \Theta' = 0$. The gains selected are $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 4$. Figure 2 shows the nondimensional tether length, the pitch angle, and the tension. It can be seen that retrieval to the desired length is completed in nearly four orbits. The gains and the initial velocity selected for this example result in an initial nondimensional tension of 4.0, which is 33% higher than the equilibrium value at the initial time. The initial velocity and the gains can be adjusted for a faster retrieval. As another example, Fig. 3 shows the retrieval process for the same initial conditions as before but with $K_1 = 1.0$, $K_2 = 0$, and $K_3 = 3$. It is clear from a comparison of Figs. 2 and 3 that by allowing higher values of the pitch angle, a faster retrieval can be accomplished. The retrieval depicted in Fig. 3 is essentially complete in less than two orbits. On the other hand, the pitch angle should not be allowed to be too high, as undesirable as well as unsafe equilibrium points may be reached. If a nonzero value of K_2 is used, the pitch angle remains small but decays very slowly.

It is also possible to hold the terminal pitch angle constant for a rapid retrieval. If the pitch angle is held constant, we notice[8] from Eq. (2a) that

$$2\lambda' = 3\lambda \cos\Theta \sin\Theta = 0 \tag{9}$$

The analytical solution to the preceding equation is

$$\lambda = \lambda_o \exp[-0.75 \sin(2\Theta)\, \lambda] \tag{10}$$

where λ_o is the initial value of λ. Therefore, the fastest retrieval rate occurs when $\Theta = 45$ deg. Furthermore, it can be shown with the help of Eqs. (2) and (10) that if the pitch angle is constant, the nondimensional tension is $3\lambda[\cos^2\Theta - 3/16 \sin(2\Theta)]$, which is positive for angles between 0 and 45 deg. With this knowledge, a terminal control law can be designed to hold the pitch angle at a small positive value. The retrieval rate is obtained first by treating it as a control variable in Eq. (2b). The tension is then determined by using the retrieval rate in Eq. (2a). If the target pitch angle is set at Θ_f, a suitable Lyapunov function is

$$V = 1/2K_1(\Theta - \Theta_f)^2 + 1/2\Theta'^2 \tag{11}$$

Carrying out the required mathematical manipulations, we obtain

$$\lambda' - \lambda[K_1(\Theta - \Theta_f) - 3 \sin\Theta \cos\Theta + K_2\Theta']/[2(1 + \Theta')] \tag{12}$$

Conclusions

A feedback scheme has been presented for the deployment and retrieval of tethered satellites. This scheme is based on tether tension control. The tension at the end of deployment is not oscillatory and is shown to reach its steady state value. It has also been shown that rapid retrieval is possible if moderate pitch angle excursion of the tether is allowed during the intermediate phase of retrieval. Fast retrieval can also be accomplished by setting a positive target pitch angle. This also ensures that the tension is always positive. These control schemes have to be further validated using more sophisticated models of the system.

Acknowledgment

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